SHAPE AND MATERIAL OPTIMIZATION OF A 2D VERTICAL FLOATING BREAKWATER

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Abstract: This paper represents an effective approach in shape and material optimization of a vertical floating breakwater. Using the potential-flow theory, the hydrodynamic pressure deduced from sea wave's propagation has been computed based on the non linear theory of Stocks. Then, an analytical structural study of the floating breakwater, exposed to the wave pressure, is elaborated based on the frame theory. Finally, the optimization process is performed on the shape and type of material of a floating breakwater; where the structural deflections and the stress distribution constitute the constraints of the optimization problem.

Keywords: Analytical modelling, wave modelling, floating breakwater, shape and material optimization.

1 Introduction

Shape optimization is a subject which has attracted the interest of the researches for many years. It refers to the optimal design of the shape of structural components and is of great importance in structural and mechanical engineering. The problem consists in finding the best shape of a structural component under certain loading, in order to have minimum weight, or uniformly distributed equivalent stresses or even to control the deflections of the structural components. A shape optimization procedure is an iterative process in which repeated improvements are carried out over successive designs until the optimal design is acceptable.

In this paper, we consider the problem of determining the optimal shape and material of a 2D vertical floating breakwater, which constitutes a new ascending type of coastal structures. These structures are designed mainly to provide protection by reflection and/or dissipation of wave energy, where rubble-mound and fixed bottom vertical breakwaters have been extensively used for sheltering harbours [1]. Nevertheless, at many locations, the site specific parameters as deep water or poor bottom conditions as well as environmental requirements including the phenomena of intense shore erosion, water quality and aesthetic considerations, advocate for the application of floating structures.

Research engineers and scientists have realized the potential for floating breakwaters in certain areas, and research interest has been directed towards this subject during the last decades due to its numerous advantages in comparison with the fixed ones. As a result many types of floating breakwaters have been developed, as described by McCartney [12]; however, the most commonly used are the rectangular pontoon-type breakwaters, which are moored to the sea bottom with cables or chains.

Moreover, many studies have been produced on floating breakwaters [13],[14] (Twu and Lee, 1983; Johansson 1989; Murali and Mani, 1997; etc.), mainly concerning the wave protection improvement by different types of floating structures. Other studies have been directed towards the mooring forces and motion responses to understand the behaviour of the floating breakwaters due to sea waves (Williams and Abul-Azm,1997; Sannasiraj, 1998; and Lee and Cho, 2003) [15], [16]. This paper seeks not only to develop a floating breakwater until that would have the capability of withstanding more severe wave loading conditions such that these structures will become a viable alternative to conventional breakwaters for moderately exposed locations, but also to optimize the shape and material of these breakwaters. This requires a comprehensive structural analysis study which constitutes the foundation of the optimization problem.

Although, the protection of marine structures has been extensively studied in recent years, understanding of their interaction with waves, marine structures and the seabed is far from complete [3]. Damage of marine structures still occurs from time to time, with two general failure modes evident. The first mode is that of structural failure, caused by wave forces acting on and damaging the structure itself. The second mode, which has attracted many of the scientists (Biot-1941; Jeng 1997; Mitzutani 1998), is that of foundation failure caused by liquefaction or erosion of the seabed in the vicinity of the structure, resulting in collapse of the structure (case of fixed bottom breakwaters only), where our work is mainly directed towards the structure failure due to the lack of knowledge in this domain. Moreover, the physical understanding and computation of wave–structure interaction, one of the most important hydrodynamic processes in both coastal and offshore engineering, are crucial to assess wave impacts on structures as well as structural responses to wave attacks. Traditionally, the estimation of wave loads on a structure is often done by either empirical approach [2] (ex: Morison equation Sainflou, Hiroi, Goda, Svendson…) or a computational approach. The empirical formulas are simple but crude and will not be able to provide detailed and accurate information about pressure distribution on a structure. The computational approach can be further divided into two types: the Laplace equation solver for potential flows [3] and the Navier–Stokes Equations (NSE) solver for viscous flows, where the latter is used for simulation of wave–structure interaction during which both vortices and turbulence may be present, where solving the Laplace equation by imposing the boundary conditions constitutes the wave modelling part in this study.

In this paper it is interesting to consider the case of a breakwater appearing in ports' constructions far from the shore, at a constant depth, and at a fixed point. Then, the problems of wave's propagation over a varying bathymetry and shallow water consequences are eliminated.

2 Wave modeling

A cartesian coordinate system *Oxyz* is employed, where *Oxy* coincide with plane of the free surface at rest, *Oz* directed positive upwards, and *Ox* directed positive in the direction of propagation of the waves. The incident wave propagates in a straight line in the direction defined by the angle γ, formed with the *Ox* axe. In this study, it is supposed that the waves can strike the breakwater in a perpendicular direction to obtain the maximum pressure applied by the waves on the breakwater. Then, the angle is taken as $\gamma = 0$ (incident wave normal to the breakwater) and the movement is reduced to two dimensions (Fig 1)

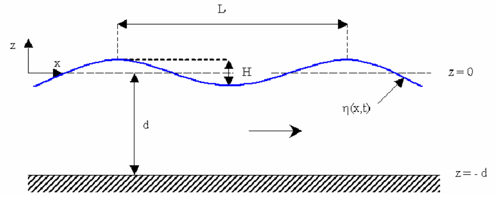


Fig.1 Wave notations

The fluid motion is defined as follows: Let t denote time, x and z the horizontal and vertical coordinates, respectively, and η the free-surface elevation above the still water level. The characteristic parameters of the wave are defined in (Fig 1). The high values of the density and sound velocity in water render the compressibility effects negligible in sea water, so it is considered incompressible. The fluid is considered also irrotational. Then, the fluid motion can be described by a velocity potential, Φ, related to the velocity $U(u, w)$.

$\overrightarrow{rot}(\breve{U}) = \breve{0} \Rightarrow \overset{\circ}{U} = \overrightarrow{grad}(\Phi)$, where $u = \frac{\partial \Phi}{\partial x}$ and $w = \frac{\partial \Phi}{\partial z}$.

Once the parameters characterizing the sea waves are known (Length of wave L, Period T, Height H), a model is needed to study the waves' propagations and transforms their evolution into loads on the breakwater. The well known equation, Bernoulli-Lagrange constitutes the essential equation to determine the field of wave's pressure.

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}(grad\Phi)^2 + \frac{P(x,z,t)}{\rho} + gz = Q(t) \qquad (1)$$

In general, the study of marine structures' behaviours due to waves' propagations is mostly made as part of a linear theory [4], where the interest in this paper is to orient the work towards the non linear approximation (Stokes 2nd order expansion). It is clear that if Φ is known throughout the fluid, the physical quantities (pressure and velocity) can be obtained from Bernoulli's equation. The boundary value problem is then defined as follows:

$\nabla^2 \Phi = \Delta\Phi = 0$ Laplace equation in the fluid domain;

$\left(\frac{\partial \Phi}{\partial z}\right)_{z=-d} = 0$ Condition at the sea floor;

$\left(\frac{\partial \Phi}{\partial n}\right)_{x=0} = 0$ Kinematic condition at the solid boundary;

$\left(\frac{\partial \eta}{\partial t} + \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x} - \frac{\partial \Phi}{\partial z}\right)_{z=\eta} = 0$ Kinematic condition at the free surface;

$\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left(\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right) + g\eta\right)_{z=\eta} = Q(t)$ Dynamic equation at the free surface;

The used method for the nonlinear theory (Stockes 2nd order expansion), called perturbation method [5], consists of developing the different variables into power series depending on a parameter $\varepsilon = H/L$, where the linear theory constitutes the first order yielding exact solutions only for waves with infinitesimal amplitudes.

$$\Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \varepsilon^3\Phi_3 + \dots\dots\dots + \varepsilon^n\Phi_n \qquad (2)$$

The nonlinear approximation is achieved by substituting for the first order in the perturbation series, where this expression of velocity potential describes the physical properties of the waves in the absence of any structure, but the reflection phenomenon must be taken into consideration during the collision of the waves by the breakwater. Finally, the expression of the pressure distribution (pressure at any point in the fluid domain.) in the case of wave-breakwater interaction, where all the waves are reflected by the breakwater (no diffraction or transmission) is given by: [17]

$$P(x,z,t) = -\rho g z + \mathrm{Re}\left\{ \frac{1}{2}\rho g H \frac{ch[k(z+d)]}{ch(kd)} \left[\exp i(kx-\omega t) + r\exp i(-kx-\omega t+\beta)\right] \right\}$$

$$+\mathrm{Re}\left\{ r\rho H^2\omega^2 \exp i(-2\omega t+\beta)\right\} - \frac{1}{4}\rho g H \frac{\pi H}{L}\frac{(r+1)}{sh(2kd)}\left[ch2k(z+d)-1\right]$$

$$+\mathrm{Re}\left\{ \frac{3}{4}\rho g H \frac{\pi H}{L}\frac{1}{sh(2kd)}\left[\frac{ch2k(z+d)}{sh^2kd}-\frac{1}{3}\right] \left[\exp 2i(kx-\omega t) + (r^2+r)\exp 2i(-kx-\omega t+\beta)\right] \right\} \quad (3)$$

(Where $k = 2\pi / L$ designates the wave number and ω the frequency). This repartition of the hydrodynamic pressure has a curved shape (obtained using Matlab); where its maximum is around the still water level and it decreases to zero at the top of the breakwater (with the wave height) and also decreases with water depth (Fig.2). Fixing $x = 0$ (exterior breakwater surface), and the phase angle $\beta = 0$ (vertical impermeable wall, [9]), the pressure distribution over the vertical breakwater is obtained.

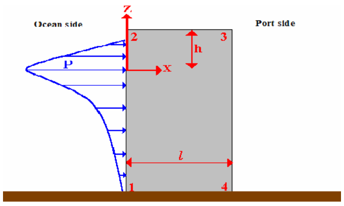


Fig. 2 Hydrodynamic pressure distribution over the breakwater

This hydrodynamic pressure is acting on the exterior surface of the breakwater due to the assumption that all the waves propagating from the ocean side are totally reflected outside the port (no transmission). It can be written as follows:

$$P = a\cosh k(z+d) + b\cosh 2k(z+d) + f \quad (4)$$

$$a = \frac{\rho g H}{2}\frac{(r+1)}{chkd}\cos(\omega t)$$

$$b = \frac{\rho g \pi H^2}{4Lsh2kd}\left[\frac{(3r^2+3r+3)\cos(2\omega t)}{sh^2kd} - r - 1\right]$$

$$f = \frac{\rho g \pi H^2}{4Lsh2kd}\left[(-r^2-r-1)\cos(2\omega t) + r + 1\right] + \rho H^2\omega^2 r\cos(2\omega t)$$

It is reduced to an equation with hyperbolic functions of z (altitude), where the other variables independent of the altitude are collected together in the terms a, b, and f. The setup for a sea wave is as flows (choosing the parameters of a strong wave): wave properties [L=140 m, T=9 sec, d=40 m, L=120 m, H=3 m, r=0.8, t=0, sea water density=1025 kg/m^3]

3 Shape optimization

A moored floating breakwater should be properly designed in order to ensure: (a) effective reduction of the transmitted energy, hence adequate protection of the area behind the floating system, (b) non-failure of the floating breakwater itself and (c) non-failure of the mooring lines. The satisfaction of these 3 requirements represents the overall desired performance of the floating breakwater. The non-failure of the mooring lines has been widely studied and discussed, so the efforts in this paper are directed towards the first two issues.

The reduction of the transmitted energy is achieved by the floating breakwater itself due to a considerable depth and by the fixed seawall concept under the breakwater for the rest underwater region. Moreover, for a breakwater to float it is obviously designed with a hollow form to reduce the total weight of the structure; where such form complicates the problem and implicates more constraints to be considered during the design.

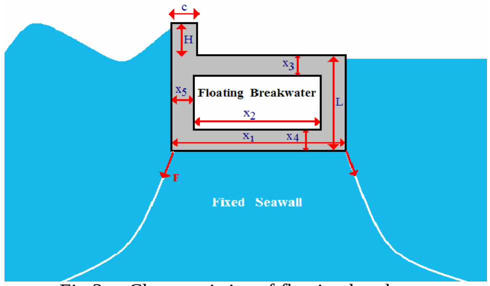


Fig.3 Characteristics of floating breakwater

An additional rectangular wall can be used to protect the sheltered regions from high waves; where it is sufficient to place it only from the ocean side since it has non sense to construct a rectangular breakwater with its height over the free surface level equals to a strong wave height. Then, it can be simply deduced that a floating breakwater can be assimilated to two parts: the main rectangular body possessing sufficient dimensions considering the fixed seawall concept, and a second part formed by a small rectangular wall fixed on the ocean side of the breakwater to attenuate the high waves. The dimensions of the second part are easily determined, where its height is equal to the wave height H, and its width c is taken to be 0.7 m [10].

Improving the performance of floating breakwaters could open up multiple of possible uses and this because the floating breakwater, in contrary to the fixed one (the only parameter to calculate is the width being deduced from the stability condition), has many parameters characterizing its geometry and defining its shape $L, x_1, x_2, x_3, x_4, x_5, F$ (Fig.3). Some of these parameters are related to the same physical constraint where the rest are determined from other independent constraints, and therefore determining its geometrical dimensions cannot be performed as an ordinary calculation problem but it needs an optimisation process in order to compute these parameters taking into consideration their effects on each other. The optimisation problem is assumed to be finite dimensional constrained minimization problem, which is symbolically expressed as:

Find a design variable vector x ;

to minimize the weight function $f_{ob}(x)$

subject to the n constraints $f_i(x) < 0$

3.1 Objective Function

The optimal solution is to design a breakwater respecting all the constraints with a minimum volume, hence the objective is to minimize the weight of the breakwater,

$$f_{ob}(x_1, x_2, x_3, x_4, x_5, F) = Lx_1 - x_2(L - x_3 - x_4) + Hc$$

3.2 Dynamic Pressure Constraint

The concept of the fixed seawall permits to determine the height of the breakwater in accordance with low hydrodynamic pressure acting on this seawall. The dynamic wave pressure is mainly concentrated near the free surface and its induced perturbation is low under a certain height (Fig.4); then the height of the breakwater can be limited to where the pressure is approximately unvarying corresponding to an approximate value of $P - 0.1P\max = 0$, where $P\max = P(z = 0)$. Finally, the height can be considered to be $L = 8m$, where this height is indeed satisfactory for a strong wave $(H = 4m)$.

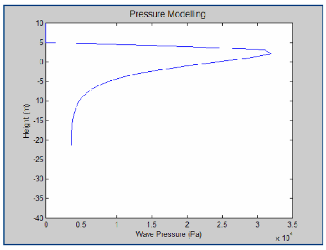


Fig.4 Wave Pressure Modelling

This constraint is independent of the other constraints, and then the height of the breakwater is determined only from it and no need to still consider the height as a variable for the rest of the optimization process.

3.3 Floating Constraint

The forces acting on the floating breakwater are numerous (Fig.5) and of various sources thus they are defined as follows:

P_1=hydrostatic pressure acting on the two sides,

P_2=hydrostatic pressure acting on bottom surface,

P_3=hydrodynamic pressure acting from the ocean side,

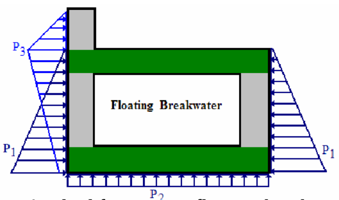


Fig.5 Applied forces on a floating breakwater

P_3 is modelled in the structural analysis as two triangular forces where the maximum is located at the height z=0 (water free surface) and it is evaluated by substituting the value of z in the equation of hydrodynamic pressure (Eq.4) , $P_3\max = a\cosh(kd) + b\cosh(2kd) + f$

The equilibrium equation for floating can be written as: $-\rho_m(V_m + V_r)g + \rho_e V_T g = 0$, where ρ_m and ρ_e designates the densities of the material (concrete) and the sea water respectively, V_m designates the volume of the inside material of the whole breakwater without the upper rectangular wall, V_o designates the volume of the hollow part (atmospheric pressure inside), V_r designates the volume of the upper rectangular part, where V_T designates the volume of the submerged part of the breakwater , and then $V_m + V_o = V_T$

A relation between the hollow volume and the submerged volume can be simply deduced:

$$V_o = \frac{\rho_m - \rho_e}{\rho_m} V_T + V_r$$

The floating constraint can be expressed as follows:

$$f(x_1, x_2, x_3, x_4, x_5, F) = x_2(L - x_3 - x_4) - \frac{\rho_m - \rho_e}{\rho_m} Lx_1 - V_r$$

But, really the floating constraint yields to a simple relation between the variables that can be used to reduce the number of variables in the optimization.

$$x_1 = \frac{\rho_m[x_2(L - x_3 - x_4) - V_r]}{(\rho_m - \rho_e)L}$$

3.4 Stability Constraint

Stability is defined as the ability of the breakwater to right itself after being heeled over. This ability is achieved by developing moments that tend to restore the breakwater to its original condition. There are a number of calculated values that together determine the stability of a floating breakwater: 1- Initial horizontal equilibrium, 2- Heeled angle, 3- Tension in mooring lines.

First of all, this floating breakwater has a non-symmetrical shape, so initially (before any disturbance) it is necessary to maintain a horizontal equilibrium position. This is performed by dividing the breakwater into 5 rectangles and calculating the new position of the centre of gravity (Fig.5) in terms of the variables and then aligning it with the centre of buoyancy for the floating breakwater (Fig.6) which lies at the geometric centre of volume of the displaced water ($x_1/2$).

$$\bar{x} = \frac{(L - x_3 - x_4)\left[\frac{(x_1 - x_2 - x_5)^2}{2} + \left(x_1 - \frac{x_5}{2}\right)x_5\right] + \frac{x_1^2}{2}[x_4 + x_3] + Hc\left[x_1 - \frac{c}{2}\right]}{x_1x_4 + x_1x_3 + Hc + (L - x_3 - x_4)(x_1 - x_2)}$$

$$\bar{y} = \frac{\left[(L - x_3 - x_4)\frac{(L - x_3 + x_4)}{2}\right](x_1 - x_2) + x_1\left[\frac{x_4^2}{2} + \left(Lx_3 - \frac{x_3^2}{2}\right)\right] + Hc\left[L + \frac{H}{2}\right]}{x_1x_4 + x_1x_3 + Hc + (L - x_3 - x_4)(x_1 - x_2)}$$

When the breakwater is disturbed by a wave, the centre of buoyancy moves from B to B_1 (Fig.6) because the shape of the submerged volume is changed; then the weight and the buoyancy force form a couple capable to restore the breakwater to its original position.

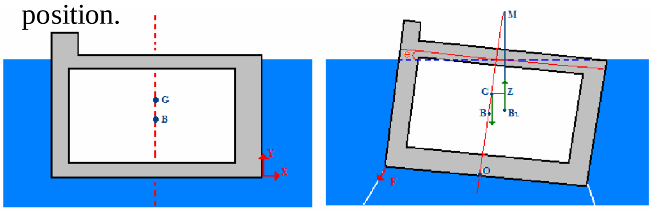


Fig.6 Stability of floating breakwater

Moreover, the distance GM known as the metacentric height illustrates the fundamental law of stability, where it must be always positive to create a restoring couple and maintain stability. The equation of motion can be written as: $\sum M = I\ddot{\theta} \Rightarrow$ at equilibrium $Mp - M_F - M_B = 0$, where Mp is the moment of the disturbing force (wave), M_F is the moment of the tension in the mooring lines, and M_B is the moment of the buoyant fore (restoring couple), the stability constraint can be expressed as

$$f_1(x_2, x_3, x_4, x_5, F) = -Lx_1\rho g\left(\frac{x_1^2}{12L} - \bar{y} + \frac{L}{2}\right)\sin\theta - F\cos(\alpha - \theta)\frac{x_1}{2}$$
$$+ F\sin(\alpha - \theta)\bar{y} + \int_0^{L/2}(a\cosh k(z + d - L + \bar{y}) + b\cosh 2k(z + d - L + \bar{y}) + f)zdz$$
$$- \int_{-L/2}^{0}(a\cosh k(z + d - L + \bar{y}) + b\cosh 2k(z + d - L + \bar{y}) + f)zdz$$

α being the angle formed by the mooring lines and the vertical (α=*20*°), and θ is the angle of disturbance (heeled angle); in fact it is fixed by the designer, and since the breakwater must be very rigid and stable in order to protect the ports from waves, it is taken to be *1.2*°.(slope of 2%)

The second relevant stability constraint is $\bar{x} = x_1/2$ (horizontal equilibrium condition)

$$f_2(x_2, x_3, x_4, x_5) = -x_1^2(x_4 + x_3) + Hcx_1 + x_1(L - x_3 - x_4)(x_1 - x_2)$$
$$2(L - x_3 - x_4)\left[\frac{(x_1 - x_2 - x_5)^2}{2} + \left(x_1 - \frac{x_5}{2}\right)x_5\right] + 2x_1^2[x_4 + x_3] + 2Hc\left[x_1 - \frac{c}{2}\right]$$

3.5 Structural constraints

This constraint constitutes a pure structural analysis of the floating breakwater, where a comprehensive structural study is requested in order to determine the bending moments, stresses, and deflections that must be restricted to certain limits. The floating breakwater is modelled as a frame structure fixed on two simple supports at its bottom, where it can be simply divided into four beams with assimilating the upper rectangular wall as a concentrated force on the upper beam. Each beam is equilibrated by the internal reactions and moments generated from frame division, and hence the equilibrium conditions can be applied for each beam alone to determine the internal efforts and moments yielding to the deflection and stress calculations, $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$. All the forces are distinguished from each other by different colours (Fig 7).

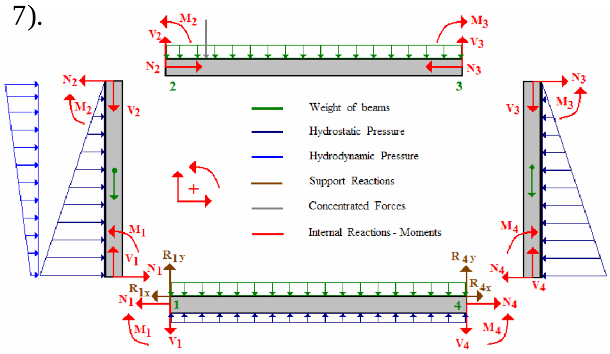


Fig. 7 Forces and moments distributions

This constitutes a problem of 12 variables (Ni, Vi, and Mi where $i = 1,2,3,4$) with 12 equations, but in fact there is only 9 effective equations (equilibrium conditions for beam 1-4, 1-2, 2-3) and the last 3 equations (beam 3-4) are linearly dependant and will not help to solve the system of 12 variables.

This problem is of the hyper-elastic type, where the number of equations is not sufficient to determine the corresponding variables [11], and it is necessary to include three other relations deduced from applying Castigliano's theorem on the fixed nodes (beam 1-4 and 1-2). $\lambda_i = \frac{\partial W}{\partial F_i} = \int M\frac{\partial M}{F_i}\frac{dx}{EI}$, λ_i being the displacement of the node where the force F_i is applied, and M the distribution of moment along the beam. Applying the global equilibrium conditions for the whole frame: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$, the support reactions $R_{1y}, R_{4y}, (R_{1x} - R_{4x})$ are expressed in terms of the variable vector *x*.

Applying the local equilibrium conditions for each of the beams 1-2, 2-3, 1-4, we obtain 9 equations in terms of the normal forces (N_1, N_2, N_3, N_4), shear forces (V_1, V_2, V_3, V_4), and the moments (M_1, M_2, M_3, M_4). Then, Castigliano's theorem is applied in beam 1-4 on the node1 and on the node 4, and beam 1-2 on the node 1; which give 3 new equations to complete the system.)

Finally, it ends up with a system of 12 variables with 12 equations, where these 12 variables (Ni, Vi, and Mi) are determined in terms of the breakwater geometrical dimensions x_1, x_2, x_3, x_4, x_5.The next step in this structural part, after determining the internal efforts and moments, is to develop the expressions of the bending stresses, and the deflections, in order to present them as new constraints needed to be respected in design. It can be easily deduced based on the following relation:

$EIy'' = M(x)$, where y'' is the second derivative of the beam deflection, *E* is the Young Modulus of the inside material, *I* is the moment of Inertia of the beam.

$\sigma = \frac{Me}{2I}$, where *e* is the beam thickness

The deflections' constraints are expressed as follows:

$$f_3(x_2,x_3,x_4,x_5)=\frac{1}{EI_{23}}[-\frac{\rho_m g x_3}{24}x^4+\frac{V_2}{6}x^3-\frac{M_2}{2}x^2-\frac{\rho_m g c^2 H}{4}x^2$$
$$-\frac{M_2 x_1}{2}x-\frac{v_2 x_1^2}{6}x+\frac{\rho_m g x_3 x_1^3}{24}x+\frac{\rho_m g c^2 H x_1}{4}x]$$
$$f_4(x_2,x_3,x_4,x_5)=\frac{1}{EI_{14}}[-\frac{(\rho_m x_4-\rho L)g}{24}x^4-\frac{V_1}{6}x^3+\frac{R_{1y}}{6}x^3+\frac{M_1}{2}x^2$$
$$-\frac{M_1 x_1}{2}x+\frac{V_1 x_1^2}{6}x+\frac{(\rho_m x_4-\rho L)g x_1^3}{24}x-\frac{R_{1y}x_1^2}{6}x]$$
$$f_5(x_2,x_3,x_4,x_5)=\frac{1}{EI_{12}}\left[\frac{\rho g}{120}(1-\frac{H}{L})y^5-\frac{\rho g L}{24}y^4-\frac{N_1}{6}y^3-\frac{M_1}{2}y^2\right]$$

The bending stresses' constraints are expressed as follows:

$$f_6(x_2,x_3,x_4,x_5)=\left[M_1-V_1x-(\rho_m x_4-\rho L)g\frac{x^2}{2}+R_{1y}x\right]\frac{x_4}{2I_{14}}$$
$$f_7(x_2,x_3,x_4,x_5)=\left[-M_2+V_2x-\rho_m g x_3\frac{x^2}{2}-\rho_m gH\frac{c^2}{2}\right]\frac{x_3}{2I_{23}}$$
$$f_8(x_2,x_3,x_4,x_5)=\left[\begin{array}{l}-M_1-N_1y-\frac{(a\cosh(kd)+b\cosh(2kd)+f)y^3}{6L}\\ -\frac{\rho g y^3}{3}-\frac{\rho g(L-y)y^2}{2}\end{array}\right]\frac{x_5}{2I_{12}}$$

All the constraints are expressed in long and complicated equations in terms of the four geometrical parameters x_2,x_3,x_4,x_5, characterising the floating breakwater. Finally, the optimization problem is summarized as follows:

Objective function:

$$Min\ \ f_{ob}(x_2,x_3,x_4,x_5)=Lx_1-x_2(L-x_3-x_4)+Hc$$

Constraints:

$$\begin{cases} f_1(x_2,x_3,x_4,x_5,F)=0, & f_2(x_2,x_3,x_4,x_5)=0\\ Max(f_3(x_2,x_3,x_4,x_5))<0.01m, & Max(f_4(x_2,x_3,x_4,x_5))<0.01m\\ Max(f_5(x_2,x_3,x_4,x_5))<0.01m, & Max(f_6(x_2,x_3,x_4,x_5))<3MPa\\ Max(f_7(x_2,x_3,x_4,x_5))<3MPa, & Max(f_8(x_2,x_3,x_4,x_5))<3MPa \end{cases}$$

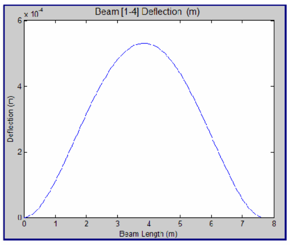
Aside from the constraints of stability, structural, and floating, it was also necessary to establish some additional geometrical constraints:

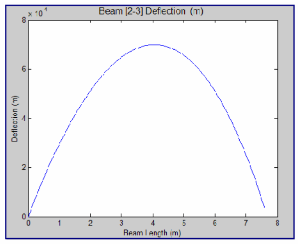
$$\begin{cases} x_2-x_1<0, \quad x_3-x_4<0\\ -x_1<0, \quad -x_2<0, \quad -x_3<0, \quad -x_4<0, \quad -x_5<0, \end{cases}$$

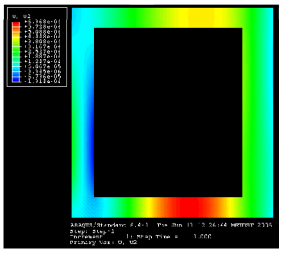
Using the Matlab optimization toolbox and mainly the function ***fmincon***; which is based on the SQP method (sequential quadratic programming), the problem can be solved to determine the variables x_2,x_3,x_4,x_5,F .

$$\begin{cases} x_1=7.68m, \quad x_2=5.65m, \quad x_3=0.8m,\\ x_4=0.8m, \quad x_5=0.845m, \quad F=1.8\times10^5\,N/1m \end{cases}$$

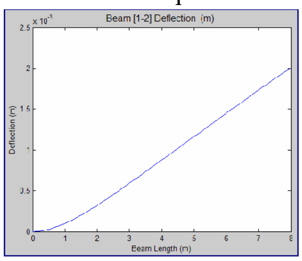
In order to validate this analytical calculation, a comparison is realized with a numerical approach using the ABAQUS software. The comparison comprises the deflections and the bending stresses of the beams (1-2, 2-3, and1-4). Using the Matlab, all the preceding equations (moments, deflections, stresses) are programmed to yield to the explanatory curves defining the real state of the floating breakwater when exposed to sea waves. First, the upper beam (2-3) is compressed from the two sides, by the hydrostatic pressure from both sides and a hydrodynamic pressure from one side, yielding to an upward deflection of *6.3x10^{-4}m*. The same trace has been drawn by ABAQUS with a close maximum deflection of *5x10^{-4}m*. The lower horizontal beam (1-4) is supporting all the weight and also the hydrostatic pressure applied at its bottom which is strong enough to cause an upper deflection *5.6x10^{-4}m* towards the hollow section and very close to the results given by ABAQUS *6.2x10^{-4}m*, except for the position of the upward maximum deflection where it is approximately located in the middle of the beam in our analytical calculation and shifted smoothly towards the right in ABAQUS.

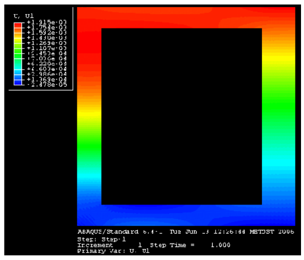




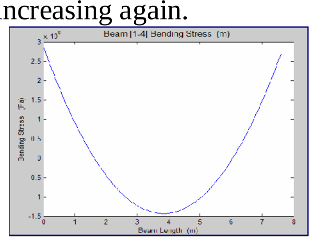


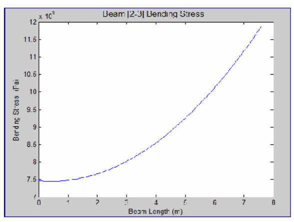
For the vertical beam (1-2), it is fixed from it lower end, and left free from above reaching a horizontal deflection of *2x10^{-3}m*, due to the hydrodynamic pressure; where the Abaqus shows a deflection of *1.9x10^{-3}m.*

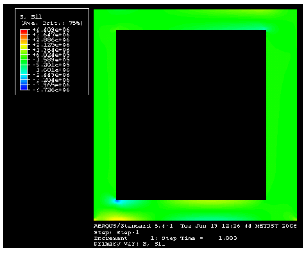




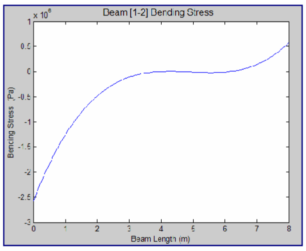
Concerning the bending stresses, a similar comparison to the deflection is realized. The bending stress in the upper beam (2-3) is increasing from *0.75 MPa* on the left side to a value of *1.2MPa* on the right side where it is reaching *1.36 MPa* in ABAQUS. For the lower beam it is decreasing from *2.8 MPa* to a minimum of *-1.45 MPa* and then increasing again. In ABAQUS, we can notice that the bending stress is decreasing from *3.64 MPa* to *-1.68 MPa* and then increasing again.

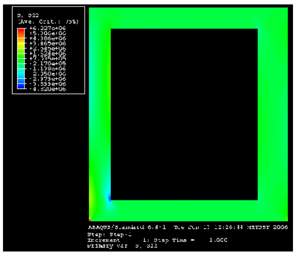






It is also showing good agreement for the bending stresses in the vertical beam (1-2).





4 Material optimization

Minimum weight of structures is not only restricted to shape optimization, but moreover to material optimization. Finding the best material type that can withstand all the exerted mechanical constraints constitutes an important step on the road of designing an optimal floating breakwater. The above analysis is performed by considering, the most utilized material in the field of marine structures, the "concrete". In this part, a comparison will be made between the concrete and other materials aiming to search for a more suitable material that can be applicable for our case.

4.1 Aluminium

Aluminium is a high quality material and also a corrosion-resistant, where it is mainly used in aircraft and ships construction due to its weight lightness and its flexibility in performing complex shapes.

The basic physical and mechanical properties are as follows:

$$\begin{cases} \text{Density } \rho = 2700\ Kg/m^3 \\ \text{Elasticity Module } E = 6.5\times10^4\ MPa \\ \text{Tensile strength } \sigma = 50\ MPa \end{cases}$$

Replacing the mechanical properties of Aluminium in the equations describing the breakwater deflections, moments, and stresses and then repeating the optimization procedure, it ends up in the following:

$$\begin{cases} x_1 = 7.29m,\ x_2 = 5.5m,\ x_3 = 0.24m, \\ x_4 = 0.8m,\ x_5 = 0.72m,\ F = 1.8\times10^5\, N/1m \end{cases}$$

4.2 Steel

Steel is a very resistant material where it can be benefited from its high mechanical properties. The basic physical and mechanical properties are as follows:

$$\begin{cases} \text{Density } \rho = 7850\ Kg/m^3 \\ \text{Elasticity Module } E = 2\times10^5\ MPa \\ \text{Tensile strength } \sigma = 200\ MPa \end{cases}$$

The calculations proved that steel cannot respect the mechanical constraints subjected to the problem of a floating breakwater, and this is clearly noticed from its great weight (high density). Because of its great weight, a floating breakwater constructed from steel must reserve a hollow part of 86 % with respect to total structure, and which is really a high ratio resulting in thin beams in the breakwater that cannot respect the deflection constraints at all. In this manner, the steel is totally excluded from the domain of materials that can be applied for a floating breakwater.

4.3 Composite materials

The accumulated experience proved that the employing of composite materials permit, with equal performance, a gain of mass varying from 10 % to 50 % over the same component in metal alloys, and with a cost of 10% to 20% less. The following properties are given for a composite material fabricated from glass/epoxy:

$$\begin{cases} \text{Density } \rho = 1700\ Kg/m^3 \\ \text{Elasticity Module } E = 12.4\times10^3\ MPa \\ \text{Tensile strength } \sigma = 90\ MPa \end{cases}$$

Repeating the same optimization procedure, it ends up with the following results:

$$\begin{cases} x_1 = 7.47m,\ x_2 = 3.73m,\ x_3 = 0.27m, \\ x_4 = 0.8m,\ x_5 = 1.6m,\ F = 1.8\times10^5\, N/1m \end{cases}$$

It is noticed that we obtained unsatisfactory results, where the frame theory is not applicable anymore (very thick vertical beams: beam 1-2, x_5=1.6m; beam 3-4, e=2m). In fact this is due to the lightness of the composite materials yielding to a low percentage of the hollow part with respect to the total structure (40 %). Due to the many advantages of the composite materials, this problem can be omitted by increasing this ratio to a higher value; this is done by fixing a concentrated or a uniform weight inside the hollow part. The floating equation is then written as follows:

$V_o = \frac{\rho_m - \rho_e}{\rho_m} V_T + V_r + \frac{\rho}{\rho_m} V$, where V is the volume of the additional weight, and ρ is the density of a reference material, Fig.8 (for example: concrete). The additional material is distributed over the length of the hollow part and a height of 1.8 m.

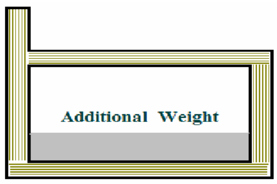


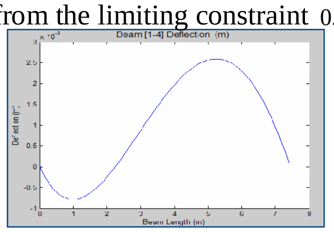
Fig.8 Floating breakwater with composite materials

It is important to note that all the beams are mainly exposed to bending effects as the Abaqus results above have proved, so the glass fibers can be placed in the direction of 0° for the horizontal beams and in the 90° for the vertical beams (Fig.8). In this type of fiber distribution, the composite material will be working in its best performance since the bending is a combination between traction for the upper part and compression for the lower part or vice versa.

Applying the optimization procedure, after increasing the ratio of the hollow part (decreasing the percentage of filled material part), we end with the following results:

$$\begin{cases} x_1 = 7.43m, \ \ x_2 = 5.83m, \ \ x_3 = 0.36m, \\ x_4 = 0.8m, \ \ x_5 = 0.63m, \ \ F = 1.8\times 10^5 N/1m \end{cases}$$

By this manner, the composite materials have proved its reliability and affectivity to replace other materials. The additional material approximated by 240 KN is modelled as a concentrated force applied on the lower beam, and then the new deflection is calculated referring to the deflection constraint f_4 (see figure below). Finally, the total deflection of the lower beam (1-4) including the additional material is still small and far enough from the limiting constraint $0.01m$.



In order to achieve the best explanation for the different materials, a comprehensive comparison (same wire tension) is elaborated in the following table:

Material	$x_1(m)$	$x_2(m)$	$x_3(m)$	$x_4(m)$	$x_5(m)$	$V(m^3)$
Concrete	7.68	5.65	0.8	0.8	0.85	27
Aluminum	7.29	5.5	0.24	0.8	0.72	22.3
Composite	7.43	5.83	0.36	0.8	0.63	21.7

As demonstrated in the table above, the composite materials seem to have the optimum choice over the rest of materials. This is not only restricted by possessing the minimum material volume, but also to the additional weight that it can hold in its hollow part. This additional material, approximated by 240 KN, can be any stocking goods and materials for the need of the ports instead of the considered reference material.

5 Conclusion

In this paper, a comprehensive study was performed towards realizing a floating breakwater that can attenuate and withstand strong waves similarly to a fixed bottom breakwater and it ended up by satisfying results. Also, an optimization process is performed to attain an optimal shape for the design of floating breakwaters. As deduced from the optimization problem, minimum weight of structures is not only restricted to shape optimization, but moreover to material optimization leading to select the composite materials from the rest. In fact, it is not only a problem of volume consuming, but also an extra load that may be used for needs storage and other used equipments in the case of the composite materials.

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